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Information dissemination by insiders in equilibrium☆

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Abstract

This paper generates an equilibrium explanation for partial disclosure of information by an insider to privileged associates. In our model, prices are set by competitive market makers in anticipation of trading volume, but not affected by the actual number of trades. Liquidity demand is not perfectly inelastic, but rather liquidity traders are sensitive to trading costs through a reservation price. Because profits from liquidity traders are bounded, the feasibility of an equilibrium depends on the balance between the number of associates, the precision of information and the number of liquidity traders. Partially, rather than fully, disclosing information alters this balance by limiting the informational advantage of *individual* associates. If the number of associates is exogenous, partial disclosure prevents market failure. If the insider chooses the number of associates, partial disclosure allows him to serve more associates but still increase total associate profits.

JEL classification: D4; D82

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1. Introduction

Selective disclosure of important information by corporate executives to institutional investors and favored analysts has long been a widespread practice.

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The Securities and Exchange Commission writes "in practice, issuers [of information] also retain control over the audience and forum for some important disclosures. If a disclosure is made at a time when no Commission filing is immediately required, the issuer determines how and to whom to make its initial disclosure. As a result, issuers sometimes choose to disclose information selectively—i.e., to a small group of analysts or institutional investors—before making broad public disclosure by a press release or Commission filing". This leaking of information to privileged market professionals can give these traders a significant advantage over uninformed traders and market makers.

This paper discusses the information dissemination strategies of corporate executives (insiders). The model is an extended version of Admati and Pfleiderer (1989). There is an insider, associates of the insider (institutional investors or analysts, for example), uninformed liquidity traders, and competitive market makers. Each associated investor trades a fixed number of shares. In equilibrium, the market makers' expected profits from trading with liquidity traders must offset the expected losses from trading with informed traders. However, the liquidity traders are sensitive to trading costs, i.e., unwilling to absorb unbounded losses. Therefore, the feasibility of a trading equilibrium depends on the balance between the number of associates, the precision of their information and the number of liquidity traders. If too many associates have information that is too precise, trading commissions do not exist which allow the market makers to break even. Other models in which information asymmetries may induce market failures include Glosten and Milgrom (1985), Glosten (1989), and Bhattacharya and Spiegel (1991). In Spiegel and Subrahmanyam (1992), a linear equilibrium does not exist when informed trading activity is too great relative to uninformed hedging activity. Empirically, Bhattacharya and Spiegel (1998) document trading suspensions in the New York Stock Exchange and present evidence consistent with adverse selection problems.

First, we consider the case in which the insider provides information to an exogenously specified number of associates. If the number of associates is low relative to the number of liquidity traders, the market remains open even if the insider reveals the exact realization of his signal about firm value to his associates. If the number of associates is too high, though, market makers are unwilling to set finite bid and ask prices because of the adverse selection problem, and effectively the market shuts down. The associates would benefit from committing not to exploit the precision of the information fully, but cannot do so credibly. Therefore, the insider must devise a mechanism to reduce, but not eliminate, the information asymmetry.

Instead of revealing the exact realization of his information, the insider can divide the possible realizations into intervals and report only the interval to the associates. A report that the signal is in the highest interval means that the signal is favorable enough that buying the security produces profits on average; a lowest-interval report means that selling the security produces profits on average; a middle-interval report means that either buying or selling will result in losses on average. One interpretation of this scheme is that the insider, through a system of decision rules (i.e., signals informing the associates when to buy, hold or sell the stock), partially reveals his information. If the decision rules induce associates to take the same actions they would take if they could observe the information directly, they do not reduce the adverse selection problem. Therefore, some bias, or distortion, of the information is necessary.[1] Following biased signals may, depending on the trading equilibrium, occasionally result in associate trading losses ex post. That is, the insider will sometimes report a "high" signal even though a buy will result in a net loss for the associate. The occasional associate losses lessen the adverse selection problem and allow the market makers to break even. The coarseness of the information prevents the associates from identifying the losing trades ex ante and unraveling the information. Associates will heed the advice of the insider because doing so is profitable *on average*.

Another way to reduce the adverse selection problem would be for the insider to provide undistorted information to fewer associates. We relax the assumption that the number of associates is fixed and allow the insider to choose the number of associates. The determination of the number of associates depends on the relative importance in the insider's preferences of (a) a large associate base and (b) average profits his associates earn. If more associates are preferred to fewer (at least over some range), the insider will trade off the precision of information with the number of associates. Clearly, individual associate profits are reduced by less precise information. However, aggregate associate profits may not decrease. In fact, the addition of bias may allow the market maker to reduce trading commissions to the point at which they *maximize* total associate profits (by maximizing liquidity trader losses). This is not possible with full disclosure, since the only way to reduce associate profits is by increasing the trading commission, which forfeits the participation of some liquidity traders. Hence, it may be optimal for the insider to manage the adverse selection problem in equilibrium through partial disclosure rather than by limiting the number of associates.

Evidence exists to suggest that material information is selectively disclosed by issuers to securities analysts or selected institutional investors or both, before making full disclosure of the same information to the general public. Moreover, insiders' preference for a larger associate base can be supported by both economic and behavioral theories. For example, Becker (1976) demonstrates how an agent behaving like an altruist (in this case, the insider providing information to more associates) benefits because the egoistic recipients of the altruism act as if they were altruistic too (reciprocal altruism). The direct effects reducing "own" consumption are stronger for an altruist toward many persons than one, but the indirect effects are also much stronger.

In SEC v. Phillip J. Stevens, the Commission alleged that a corporate official's "personal benefit" from the disclosure arose from a desire to protect and enhance his reputation, although he did not profit directly by trading on the information. Considerable evidence in psychology suggests that to be *in the know* exalts one's self-importance and reputation, and while giving the information, the person is for the

[1] We will use *bias* and *distortion* interchangeably in the text to designate non-random perturbation of the underlying information.

time being, renowned by his listeners (see for example, Allport and Postman, 1947). Because reputation can only be enhanced if the insider provides the information to sufficiently many people, a manager's reputation concerns can lead to a preference for providing information to more, rather than fewer affiliates.[2] Further, providing information to more associates likely enhances the manager's power, social capital and in turn, wealth. In social networks, a central figure has greater power when he has links with more rather than fewer agents, especially when these agents do not communicate (as is the case with competing analysts or institutional investors). A manager benefits from the creation and maintenance of links between analysts and institutional investors as bridge relationships are often associated with more positive peer reputations and higher compensation (Burt, 2002). Thus, the sharing of information can translate into rewards tit for tat. For example, the insider might attain positions on the board of directors of the institutional investors' firms, receive additional publicity and promotion from analysts, or other forms of indirect remuneration.

Our paper is related to Glosten (1989), which, in addition to deriving theoretical conditions for market failure, also discusses institutional mechanisms to prevent it. In particular, he shows that a monopolist-specialist may be willing to open the market in situations in which competitive market makers will refuse to transact with informed traders. The monopolist-specialist, unlike competitive market makers, is not constrained to earn zero profits. Thus, a monopolist may be willing to absorb losses for some extreme values of insider information as long as the pricing schedule induces higher (monopolist) profits for more moderate realizations of private information. The advantage of the monopolist is that he can average profits and losses across trades. In this sense, the mechanism is not unlike the mechanism in our paper, though in our paper it is the informed associate who may be willing to absorb losses in some cases in order to have the opportunity to profit in expectation.

Our paper is also related to Admati and Pfleiderer (1986). They study the optimal policy of a monopolistic information seller in a rational expectations framework. Because private information is revealed through bids and offers, the value of information is decreasing in the number of traders who hold it, and the speed at which the value decreases is increasing in the precision of the information. To maximize rents (price times number of purchasers), the informed seller should add individual specific random noise to his signal before selling it. The coarse signals in our model also represent a form of noise. However, in Admati and Pfleiderer (1986) the insider provides information truthfully whereas in our model, partial disclosure allows the insider to add bias as well.

The remainder of the paper proceeds as follows. Section 2 lays out the model. Section 3 establishes the conditions necessary for a full disclosure equilibrium to exist. Section 4 discusses the implications of partial disclosure. In Section 5 we discuss the endogenous choice of the number of associates. In Section 6, we summarize the results of the paper and discuss their implications.

[2] Senior (1938) writes "the desire for distinction … may be the most powerful of human passions. All the gold in the Pactolus … would obviously confer no distinction on the man who was unable to exhibit it".

2. The model

A single infinitely lived risky asset trades in periods $1, 2, \ldots, \infty$.[3] The value of the asset is $\tilde{D}_\infty = \bar{D} + \sum_{t=1}^{\infty} \delta_t$. The individual innovations, $\tilde{\delta}_t \in [\underline{\delta}, \bar{\delta}]$, are independently distributed with density $f(\delta)$ and distribution $F(\delta)$. The density $f(\cdot)$ attains its maximum at $\delta = 0$. Furthermore, it (weakly) decreases symmetrically and monotonically on either side of zero. In period t, the realization of $\tilde{\delta}_t$ becomes public information. Accordingly, the expected value of the asset, based on publicly available data at time t, is $D_t = \bar{D} + \sum_{\tau=1}^{t} \delta_\tau$.

There are four types of risk-neutral market participants. In each period, there are (at least) two market makers, an insider, N_L liquidity buyers, N_L liquidity sellers and a finite number ($N_I < \bar{N}$) of informed associates (i.e., traders who receive information from the insider). In period t, market maker j announces a bid price, B_t^j, and an ask price, A_t^j, price at which *all* sells and buys are transacted, respectively.[4] The market maker has access to all public information when setting bid and ask prices. As in Admati and Pfleiderer (1989), we define the ask commission as the excess of the ask price over the market maker's expected value of the asset, or $a_t^j \equiv A_t^j - D_t$. The bid commission is similarly defined as $b_t^j \equiv D_t - B_t^j$. That is, a buyer pays the market maker $D_t + a_t^j$ per share, but the market maker pays a seller only $D_t - b_t^j$ per share. The trading commissions, a_t^j and b_t^j, reflect the cost to an uninformed trader of trading with market maker j in period t.

2.1. Liquidity traders

Each liquidity trader seeks to buy or sell one share in the current period for exogenously determined liquidity reasons. Unlike Kyle (1985), the liquidity traders' demands are sensitive to the market makers' trading commission. A liquidity buyer or seller has a reservation price, $\tilde{r} \in [\underline{r}, \bar{r}]$ with density function $g(r)$ and cumulative distribution function $G(r)$.[5] If $a_t^j < r$, the liquidity traders on the buy side of the market submit a buy order. If $b_t^j < r$, the liquidity traders on the sell side of the market submit a sell order. If trading costs are too high ($a_t^j > r$ or $b_t^j > r$), liquidity traders do not transact in period t, nor do they trade in any subsequent period on the liquidity demand of period t. In expectation, the market maker profits from trades with liquidity traders.

If the ask commission in period t is a_t, a liquidity buyer trades in period t with probability $\Pr(a_t < \tilde{r})$. The expected profit of a market maker trading with a liquidity buyer is a_t, the difference between the ask price and the expected value of the asset conditional on public information. Therefore,

$$L(a_t) = a_t \Pr(a_t < \tilde{r}) \tag{1}$$

[3] A glossary of notation appears Table 1.

[4] This is consistent with the models in Easley and O'Hara (1987) and Admati and Pfleiderer (1989).

[5] We also assume that $\underline{\delta} < \underline{r} < \bar{r} < \bar{\delta}$.

Table 1
Notation

Term	Definition
D_t	expected value of asset at time t based on public information
$\tilde{\delta}_{t+1}$	innovation in period $t+1$
$F(\delta), f(\delta)$	CDF and PDF of δ
N_L	number of liquidity traders
N_I	number of insider's associates
B_t^j	market maker j's bid price in period t
A_t^j	market maker j's ask price in period t
a_t^j	market maker j's ask commission in period t
b_t^j	market maker j's bid commission in period t
$\tilde{r}$	reservation value of liquidity traders
$G(r), g(r)$	CDF and PDF of $\tilde{r}$
$L(\cdot)$	liquidity trader losses
a_L^t	ask commission that maximizes $L(a)$
y^{buy}	insider's buy/hold cutoff
y^{sell}	insider's hold/sell cutoff
$H(\cdot,\cdot)$	benefit insider derives from associate base
β	insider discount rate
$I(a_t, k^A)$	buying associate's expected profits with positive bias of k^A
$I(b_t, -k^B)$	selling associate's expected profits with positive bias of k^B
$\bar{N}$	market makers' volume capacity
$\Pi_t^A(a_t, k^A)$	market maker's buy-side profits at ask a_t and bias k^A
$\Pi_t^B(b_t, k^B)$	market maker's sell-side profits at bid b_t and bias k^B
a^*	ask commission that maximizes Π^A
k^A	bias/distortion in buy/hold cutoffs
k^B	bias/distortion in hold/sell cutoffs

represents a liquidity buyer's expected losses to the market maker from trading at an ask commission of a_t. We assume that the function $L(\cdot)$ is concave and has an interior maximum, which we call a_t^L. A parallel argument holds for the sell side of the market, and $L(b_t)$ represents the expected loss of a liquidity seller trading at a bid commission of b_t. The bid commission which maximizes liquidity losses is b_t^L.

2.2. The insider

The insider receives perfect information about the current period's innovation, δ_{t+1}, and provides some version of it to his associates. In Section 3, he fully discloses the information, providing the associates with the exact realization δ_{t+1}. In Section 4, we discuss partial disclosure equilibria in which the insider provides decision rules to the associates. The decision rules are a coarse version of the underlying information. The insider strategically determines $y = [y^{\text{sell}}, y^{\text{buy}}]$, where y^{sell} and y^{buy} are the cutoffs on the real line that define the sell, hold, and buy regions. When the insider issues decision rules, rather than point estimates, the associates *cannot* perfectly infer the value of δ_{t+1}.

A partial disclosure strategy of $y^{\text{buy}} = a_t$ and $y^{\text{sell}} = -b_t$ induces the same investment choices as full disclosure of δ_{t+1}. The associate suffers no loss from the coarsification of the information per se. However, a partial disclosure strategy of $y^{\text{buy}} \neq a_t$ and $y^{\text{sell}} \neq -b_t$ is not equivalent to full disclosure. The "distorted" decision rule may induce the associates to make different trading decisions than they would make if they could observe δ_{t+1} directly. We will return to partial disclosure in Section 4 and discuss the way in which the insider distorts his information to alleviate the market makers' adverse selection problem.

We assume that the insider acts to maximize the following objective function:

$$\sum_{t=1}^{\infty} \beta^{t-1} H(N_I, I(a, k^A) + I(b, -k^B)), \tag{2}$$

where $I(a, k^A)$ and $I(b, -k^B)$ are functions defining the expected associate profits per trading round for buys and sells respectively, and β is the insider's discount rate. Let H_1 and H_2 be the derivatives of the function with respect to the first two arguments, and let H_{12} be the cross-partial. We assume that all three are positive. The insider prefers more associates to fewer, and higher expected associate profits to lower. Also, the per client benefit is increasing in the average client profits. The formulation allows for a variety of insider preferences. The insider's choice variables are the number of associates (N_I), and the cutoffs y^{buy} and y^{sell}, which are a function of the chosen distortion in signals (to be defined later).

Our objective function above suggests that insiders weakly prefer to share information with more associates than fewer, arising from expected reputation and indirect wealth benefits associated with the selective disclosures. Recall that this objective can be supported by the ideas of reciprocal altruism, reputation enhancement and strategic dependency. That is, although providing information to additional affiliates may not provide direct benefits to the insider, his actions lead his beneficiaries to behave altruistically as well. The indirect effects are that the affiliates consider the insider's welfare in their own objective functions, and thus insider's consumption can indirectly increase. A desire to maintain a good reputation and close ties with institutional investors or analysts also provides such motivation. The higher profile and/or greater power that comes as a result of sharing information, can affect the insider's wealth and job opportunities.

2.3. The insider's associates (informed traders)

Each associate of the insider may purchase or sell one share of the asset. The restriction to a finite number of shares eliminates the prospect of infinitely many orders and unbounded profits when the innovation is above (below) the ask (bid) commission. Appealing to the discussion in Admati and Pfleiderer (1989), the restriction can be loosely viewed as the result of (unmodeled) risk aversion or capital constraints.

If the insider tells the associate the realization δ_{t+1}, the associate buys if $\delta_{t+1} > a_t$. The expected profit function of an associate buyer in period t is

$$I(a_t) = \Pr(\tilde{\delta}_{t+1} > a_t)\,(\mathrm{E}[\tilde{\delta}_{t+1}|\tilde{\delta}_{t+1} > a_t] - a_t) \tag{3}$$

$$= \int_{a_t}^{\bar{\delta}} (\delta - a_t) f(\delta)\,\mathrm{d}\delta. \tag{4}$$

That is, for an ask commission a_t, the ex ante probability that an associate will purchase a unit of the risky asset is $\Pr(\tilde{\delta}_{t+1} > a_t)$. The second term in Eq. (3), $(\mathrm{E}[\tilde{\delta}_{t+1}|\tilde{\delta}_{t+1} > a_t] - a_t)$ is an associate's expected profit, conditional on the associate buying at the ask commission a_t.[6] Expected profits are decreasing in the ask commission a_t.[7]

The expected profit of an associate seller is

$$\Pr(\tilde{\delta}_{t+1} < -b_t)(-b_t - \mathrm{E}[\tilde{\delta}_{t+1}|\tilde{\delta}_{t+1} < -b_t]) = \int_{\underline{\delta}}^{-b_t} (-b_t - \delta) f(\delta)\,\mathrm{d}\delta.$$

Given the symmetry of $f(\cdot)$ around 0, we write an associate seller's expected profit, $I(b_t)$ as

$$I(b_t) = \int_{b_t}^{\bar{\delta}} (\delta - b_t) f(\delta)\,\mathrm{d}\delta.$$

2.4. Market makers

There are also at least two market makers who post prices for trade with all types of traders. We assume that the market makers have some finite capacity for processing transactions of $\bar{N}$ shares.

Total expected profits to the market maker from trading at the *ask* commission a_t are

$$\Pi_t^A(a_t) = N_L L(a_t) - N_I I(a_t). \tag{5}$$

Total expected profits to the market maker from trading at the *bid* commission are

$$\Pi_t^B(b_t) = N_L L(b_t) - N_I I(b_t). \tag{6}$$

Recall that N_L and N_I are the number of liquidity traders and informed associates respectively. Since the multiple market makers simultaneously post bid and ask prices, the Bertrand competition drives market makers' profits to zero. Moreover, if there are multiple bid/ask commissions that satisfy the zero-profit condition, competition forces the market maker to select the minimum of these trading commissions. As in Admati and Pfleiderer (1989), expected profits from trading at the ask (bid) commission for every market maker who posts the lowest ask (bid) commission are proportional to $\Pi_t^A(a_t)$, and thus $\Pi_t^A(a_t) = \Pi_t^B(b_t) = 0$ for every period t. In the first period, the market maker sets the bid and ask commissions so that expected profits are zero. The zero-profit condition and the market maker profit

[6] Because $f(\delta)$ is independent of t, we can suppress the subscript in the integral.

[7] Admati and Pfleiderer (1989) demonstrate this result.

function in Eq. (6) imply that market makers make up their losses to informed associates through gains from liquidity traders.

3. Full disclosure

In this section and Section 4, we treat N_I as exogenous and observable. In Section 5 we address the incentive issues that arise when the insider chooses N_I. We suppress time subscripts whenever possible.

If the market makers set the bid and ask commissions so that trading is prohibitively costly (even to the informed associates), there is a no-trade equilibrium. At trivial trading commissions, the market makers earn zero profits because no category of trader participates in the market. This is equivalent to market failure. Empirical evidence suggests that on average the NYSE suspends the trading of four common stocks every day, during which time the specialists do not trade and the exchange does not provide liquidity in the affected stocks (Bhattacharya and Spiegel, 1998). In the remainder of the paper, we consider actions on the part of the insider to avoid the no-trade equilibrium.

A full disclosure equilibrium is defined as follows:

Definition 1. A full disclosure trading equilibrium is set of bid and ask commissions, $m = (b, a)$ and an information disclosure strategy such that

(i) $N_L L(a) = N_I I(a)$ and $N_L L(b) = N_I I(b)$.
(ii) The insider provides the associates with the exact realization δ_{t+1}.

Condition (i) is the market makers' zero-profit condition. Condition (ii) requires the insider to disclose δ_{t+1} fully to the associates. The associates optimally buy if $\delta_{t+1} \geqslant a$, sell if $\delta_{t+1} \leqslant -b$, and hold otherwise.

Define N_I^* as

$$N_I^* = \max_a \frac{N_L L(a)}{I(a)} \tag{7}$$

$$= \max_b \frac{N_L L(b)}{I(b)} \tag{8}$$

or the maximum number of associates for which there exists an ask commission that will provide the market makers with at least zero profits.[8] Proposition 1 then follows.

Proposition 1. *For any $N_I \leqslant N_I^*$ a full disclosure equilibrium exists in which*

(i) *Market makers charge the lowest trading commissions for which they earn zero profits, or $m = [-b, a]$, where a and b are implicitly defined by $N_L L(a) - N_I I(a) = 0$ and $N_L L(b) - N_I I(b) = 0$.*
(ii) *The insider provides the associates with the exact realization δ_{t+1}.*

All proofs are in the appendix.

[8] The assumption that $\underline{\delta} < \underline{r} < \bar{r} < \bar{\delta}$ implies that finite N_I^* exists.

Observation 1. *No full disclosure trading equilibrium exists when* $N_I > N_I^*$.

If an insider fully discloses δ_{t+1} when $N_I > N_I^*$, market makers are unwilling to set commissions less than ∞. There are too many informed traders relative to the number of uninformed traders. Therefore, market makers cannot achieve zero profits.

3.1. A numerical example

A numerical example illustrates the dynamics of the model. Let $\tilde{\delta} \sim U[-10, 10]$, $\tilde{r} \sim U[0, 5]$, $N_L = 300$. Simple manipulation of the density functions reveals that $N_I^* = 300$. The columns under the heading "Full Disclosure" in Table 2 give the full disclosure equilibrium values for different values of N_I. As N_I increases, the market makers must increase the trading commissions to absorb the losses from the additional informed traders. Commissions become more extreme, and the hold region expands. We focus on the ask side of the market when $N_I = 267$. At $N_I = 267$, the market makers charge $a^L = 2.50$, liquidity traders transact with probability 0.5, and the expected profits from liquidity traders on the bid and ask sides of the market are (300)(0.5)(2.5) = 375. Associates buy, hold and sell 37.5%, 25%, and 37.5% of the time, respectively. The expected value of the asset, conditional on its value being above $a = 2.50$, is 6.25. Each associate expects to earn $6.25 - 2.50 = 3.75$ per share, and will purchase a share with probability $(1 - F(2.5)) = 0.375$. Thus, total expected profits to associates on the buy side are (267)(3.75)(0.375) = 375, which is maximal. From $a = 2.50$ to 3.33, market makers' losses to associates fall

Table 2
Numerical example: $\tilde{\delta} \sim U[-10, 10], \tilde{r} \sim U[0, 5], N_L = 300$

	Full disclosure				Partial disclosure		
Variables$\diagdown N_I$	100	200	267	300 (N_I^*)	310	310	310
a	0.840	1.745	2.500	3.333	2.500	4.649	4.649
b	0.840	1.745	2.500	3.333	2.500	4.649	4.649
k^A	—	—	—	—	−2.804	4.000	−4.000
k^B	—	—	—	—	2.804	4.000	−4.000
Buy cutoff ($y^{\text{buy}} = a - k^A$)	—	—	—	—	5.304	0.649	8.649
Sell cutoff ($y^{\text{sell}} = -b - k^B$)	—	—	—	—	−5.304	−8.649	−0.649
% Buy	45.8	41.3	37.7	33.3	23.5	46.8	6.8
% Hold	8.4	17.4	25.0	33.3	53.0	46.4	46.4
% Sell	45.8	41.3	37.7	33.3	23.5	6.8	46.8
E[δ_{t+1}|buy]	5.420	5.872	6.250	6.667	7.652	5.325	9.325
E[δ_{t+1}|hold]	8.4	17.4	25.0	33.3	46.4	46.4	53.0
E[δ_{t+1}|buy]	−5.420	−5.872	−6.250	−6.667	−7.652	−9.325	−5.325
Total associate profits	209.8	340.8	375.0	333.3	375.0	97.9	97.9
Profit per associate	2.097	1.704	1.406	1.111	1.210	0.316	0.316

sufficiently fast to offset the declining gains from lower participation of liquidity traders. However, total associate profits are diminished, falling from 375 to 333.3. If N_I exceeds 267, there is no full disclosure trading equilibrium.

4. Partial disclosure

If $N_I > N_I^*$, the adverse selection problem is too severe under full disclosure, and the market makers are unwilling to set finite bid and ask prices. In this section, we show that partial disclosure, in the form of trading rules by the insider, can recover a trading equilibrium (i.e., an equilibrium at which the ask commission is non-trivial) for $N_I > N_I^*$. We continue to treat N_I as exogenous and observable. In Section 5, we address the insider's optimal choice of N_I.

4.1. Positive distortion

Under partial disclosure, the insider coarsens the information he provides to the associates by issuing decision rules rather than the realization δ_{t+1}. The insider's strategy is $y = [y^{\text{sell}}, y^{\text{buy}}]$, where y^{sell} and y^{buy} are the cutoffs on the real line that define the sell, hold, and buy regions. As long as (i) $\mathrm{E}[\tilde{\delta}_{t+1}|\text{buy}] > a_t$, (ii) $\mathrm{E}[\tilde{\delta}_{t+1}|\text{sell}] < -b_t$, and (iii) $-b_t \leqslant \mathrm{E}[\tilde{\delta}_{t+1}|\text{hold}] \leqslant a_t$, the informed trader rationally follows the decision rules provided by the insider. Consider the associates' alternatives when receiving a buy signal. An informed associate could hold, rather than purchase the asset. If the associate holds (i.e., does not transact), he earns zero profits. However, $\mathrm{E}[\tilde{\delta}_{t+1}|\text{buy}] > a_t$, and thus an informed associate's expected profits from purchasing a share are positive. Therefore an informed associate will buy rather than hold. It is obvious that he will buy rather than randomize between buying and holding when given a signal to buy. Finally, an informed associate could sell the asset when he is given a signal to buy. Selling the asset when it has a positive expected value results in expected losses (the sum of the bid commission and the foregone profits of purchasing the asset at a price below value). Hereafter, we will only consider the insider's decision rules that will be followed by his associates, or those that satisfy conditions (i)–(iii).

In Section 2.2 we noted that a strategy of $y = [-b, a]$ induces the same investment choices as full disclosure of δ_{t+1}. Hence, coarsification alone does not reduce the adverse selection problem. Suppose that the insider bases the buy signal on the private information distorted by $k^A \geqslant 0$. The insider recommends that his associates buy whenever $\delta_{t+1} + k^A$ exceeds the ask commission a^j, implying that the *cost* of the asset is lower than the expected value of the liquidating dividend plus some distortion. Signals based on a distortion of k^A in privately observed innovations δ_{t+1} are said to be biased by k^A. The bias in decision rules is now a parameter in the associate's profit function, or

$$I(a, k^A) = \int_{a-k^A}^{\bar{\delta}} (\delta - a) f(\delta)\, \mathrm{d}\delta.$$

When $k^A = 0$, the insider's signals are unbiased. As before, the associate can calculate the expected value of the asset given the signal but cannot determine the information δ_{t+1} precisely because k^A does not depend on δ_{t+1}. Under partial disclosure, the decisions based on the signal the insider provides may differ from the decisions the associates would make if they observed δ_{t+1} directly. Positive distortion creates a region, $a - k^A \leqslant \delta_{t+1} < a$, in which the decision rule results in trades that are too "optimistic". As a result, associates will lose on some trades when signals are positively distorted. For an example, see Fig. 1, panel A. The associates buy whenever $\delta_{t+1} > a - k^A$. Therefore, in the area labeled "region of over-buying", the associates buy even though the next period's innovation is less than the current period's ask commission. Of course, the associates do not know whether they are in the region of overbuying, or the region of "correct" buying, since they cannot perfectly determine δ_{t+1} from the signal.

Observation 2. *Given the ask commission a, $\Pi^A(a, k^A)$ is increasing in k^A.*

Proofs of observations are given in the appendix.

Observation 3. *There exists k^A such that $\Pi^A(a, k^A) = 0$.*

A positive distortion of $a - \underline{\delta}$ implies that the insider always tells the associate to buy. Since the expected value of $\delta_{t+1} = 0$, the associates break even on the asset but always pay commission of a. As $k^A \to a - \underline{\delta}$, associates' expected profits converge to $-a$. Therefore, there exists a level of distortion that drives associate profits arbitrarily close to zero. Recall, the associates follow the decision rule because their expected profits are greater than zero. They observe only the buy signal, not δ_{t+1} or $\delta_{t+1} - k^A$. Hence, they cannot invert and thereby "undo" the distortion.[9]

With a positive distortion on the sell side, the insider issues a sell signal if $\delta_{t+1} + k^B < -b$ (Fig. 1, panel A). With positive bias, the associate's expected sell profit is

$$\int_{\underline{\delta}}^{-b-k^B} (-b - \delta) f(\delta)\, \mathrm{d}\delta,$$

which, by symmetry, is equivalent to $\int_{b+k^B}^{\bar{\delta}} (\delta - b) f(\delta)\, \mathrm{d}\delta = I(b, -k^B)$. Since the market maker's expected loss to a sell associate is $I(b, -k^B)$, positive distortion leads associates to forego profitable trades in equilibrium in region $-b - k^B \leqslant \delta_{t+1} < -b$. This is the region labeled "underselling" in Fig. 1.[10] However, all trades in which they do engage are profitable. As before, market maker bid-side profits of $\Pi^B(b, k^B)$ are increasing in k^B. Finally, an existence result similar to Observation 3 holds.

Observation 4. *There exists k^B such that $\Pi^B(b, k^B) = 0$.*

[9] The insider can also lower associate profits by adding negative distortion to signals. This analysis runs parallel to the above analysis and is suppressed.

[10] They do not know which trades would be profitable ex ante, however.

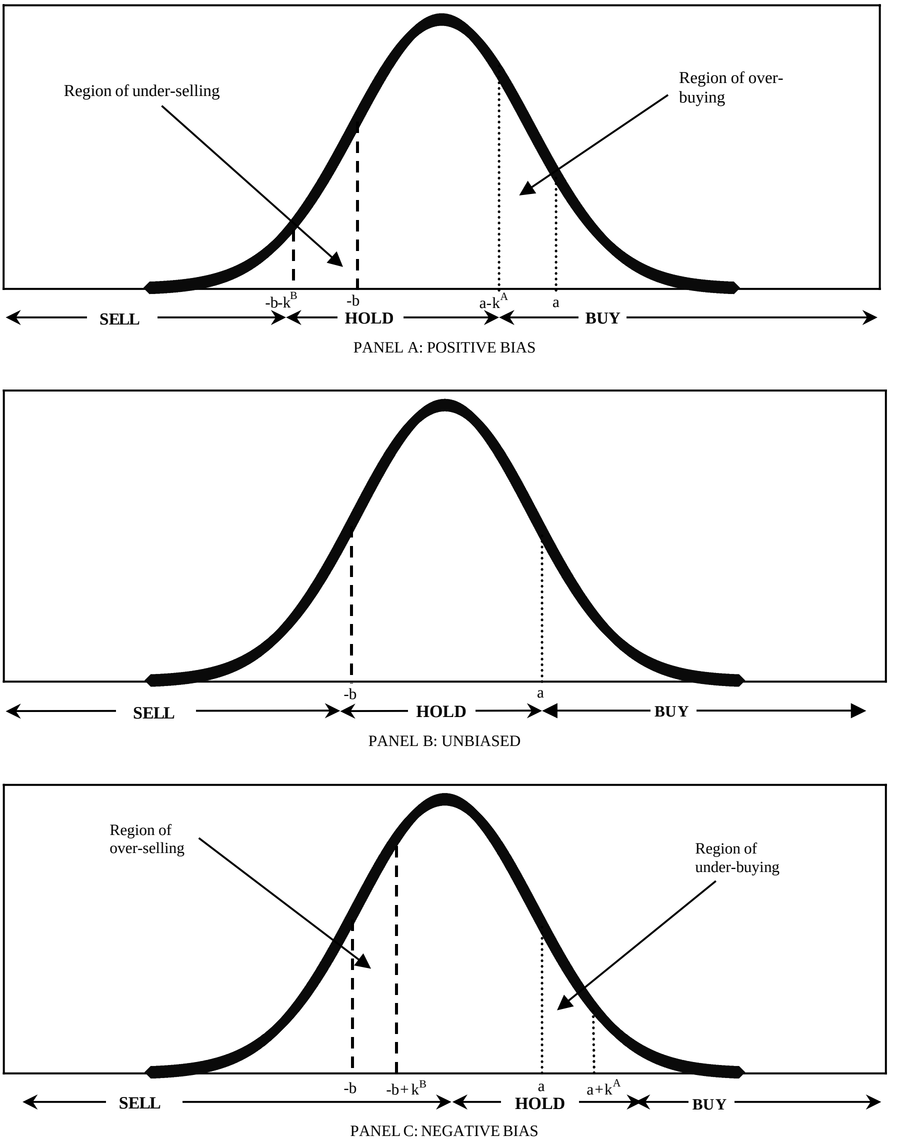

Fig. 1. The effects of recommendation bias.

4.2. Partial disclosure equilibria

Observations 2–4 demonstrate that the introduction of distortion of k^A into the insider's decision rules can drive $\Pi^A(a, k^A)$ to zero for arbitrary a and N_I. Hence, a *partial* disclosure equilibrium for N_I may be possible even when a full disclosure equilibrium is not.

We define a partial disclosure trading equilibrium below.

Definition 2. A partial disclosure trading equilibrium is a set of bid and ask commissions, $m = (b, a)$, a strategy y which is a function of distortions $k^A > 0$, $k^B > 0$ and market makers' beliefs, κ^A and κ^B such that

(i) $N_L L(a) = N_I I(a, \kappa^A)$ and $N_L L(b) = N_I I(b, -\kappa^B)$. The market makers' beliefs are confirmed in equilibrium ($\kappa^A = k^A$ and $\kappa^B = k^B$) and are formed conditional on the distortion in the previous period.
(ii) The insider divides the real line into three intervals such that the insider advises his associates to buy if $\delta_{t+1} \geqslant a - k^A$, sell if $\delta_{t+1} \leqslant -b - k^B$, and hold otherwise.
(iii) $\mathrm{E}[\tilde{\delta}_{t+1}|\text{buy}] \geqslant a$; $-b < \mathrm{E}[\tilde{\delta}_{t+1}|\text{hold}] < a$; $\mathrm{E}[\tilde{\delta}_{t+1}|\text{sell}] \leqslant -b$.

The first condition ensures that the market makers receive zero expected profits in equilibrium and the beliefs of the market makers about the amount of distortion in the insider's advice are confirmed on the equilibrium path. The second condition eliminates overlapping regions. The last condition guarantees that the associate acts rationally in following the insider's advice. That is, the associate expects ex ante to profit from following the buy and sell signals he receives from the insider. Furthermore, the associate's optimal action if the insider advises hold is neither to buy nor sell.[11] In principle, strategies entailing noncontiguous recommendation regions are possible. We discuss this possibility in the next section, but limit the main analysis in the paper to insider strategies with contiguous signal regions.

By issuing the coarse decision rule, rather than the realization of δ_{t+1}, prior to trading, the insider allows associates to commit to a less profitable trading strategy. If the associates knew δ_{t+1} prior to the trading date, they could not credibly commit to making ex ante unprofitable decisions; they would renege in the region of "overbuying" or "underselling." Since a full disclosure equilibrium, equivalent to revealing δ_{t+1}, cannot exist for $N_I > N_I^*$, the coarsification of information in the form of buy/hold/sell regions arises endogenously.

Different combinations of positive or negative distortions to the insider's information produce different patterns of skewness in the decision rules, as shown in Table 3. Positive distortion in signals decreases both the buy/hold and the hold/sell cutoffs (y^{buy} and y^{sell} respectively). Negative distortion in signals increases both y^{buy} and y^{sell} (panel C of Fig. 1).

We focus on a symmetric Reduced Trade equilibrium in Proposition 2 below. In this equilibrium, the insider advises his associates to trade only for extreme realizations of the information. The cutoffs for buys and sells are equally extreme. Unlike the other equilibria, associates never make losing trades in the Reduced Trade equilibrium. That is, the insider limits the losses of the market makers by excluding his associates from some profitable trades rather than by involving them in

[11] We note that the "players" in the market do not compete in the standard way. That is, the market maker has a strategy and beliefs (i.e., forms expectations), but is competing with other market makers (which we suppress), not the associates.

Table 3
Signal distortion and skewness

	Buy (+ distortion)	Buy (– distortion)
Sell (+ distortion)	Skewed buy	Reduced trade
Sell (– distortion)	Reduced hold	Skewed sell

some unprofitable ones. The proof of the existence of equilibria in which associates occasionally enter ex ante unprofitable trades is similar to the proof of the Reduced Trade equilibrium.[12]

Proposition 2. *A subgame perfect symmetric Reduced Trade partial disclosure equilibrium exists for* $N_I > N_I^*$ *if the insider's discount rate* β *is sufficiently high. More specifically, consider* $N_I \in (N_I^*, \bar{N}]$. *Let* a^* *and* b^* *maximize the market makers' expected profits with no insider distortion.*

(i) *The market makers charge commissions* $m = (b^*, a^*)$ *as long as the order flow in period* $t-1$ *does not imply that the insider failed to bias buy signals by at least* $-k^A$ *or failed to distort sell signals by at least* k^B. *Otherwise, the market makers set the trading commissions to* ∞.[13]

(ii) *The insider's decision rule cutoff strategy is* $y = [(-b^* - k^B), (a^* + k^A)]$. *The ask distortion,* $-k^A$, *is implicitly determined by* $N_L L(a^*) = N_I I(a^*, -k^A)$; *the bid distortion,* k^B *is determined by* $N_L L(b^*) = N_I I(b^*, -k^B)$.

After the trading round, the market makers can observe both the order flow and the true value of δ_{t+1}. If the order flow implies that the insider has provided advice inconsistent with the required level of distortion in δ_{t+1}, the market makers will shut down the market (i.e., set the bid and ask commissions in the interval $[\bar{\delta}, \infty)$) in every future period. The most profitable ask-side defection for the insider is for a realization of $\delta_{t+1} = a^* + k^A$, in which case he advises buy instead of hold. Current period benefits are higher, but future period benefits, because of the possibility of market shutdown, are lower. Equating the defection and non-defection expected utilities and solving for β provides the discount rate necessary for the proposition. An identical argument holds for the bid side of the market. Implicitly, then, the beliefs of the market are as follows: the expected levels of buy and sell side bias in the current period are $-k^A$ and k^B as long as the bias in the previous period equals or exceeds $-k^A$ and k^B. Otherwise the market maker assumes the insider will provide unbiased disclosure rules.

[12] Attaining a cooperative solution with multiple insiders might also be possible under an application of the Folk Theorem.

[13] *Any* trade commission above $\bar{\delta}$ effectively shuts the market down.

4.3. The numerical example, continued

Refer back to Table 2 under the heading "Partial Disclosure". If $N_I > 300$, the insider must distort his signals to maintain a non-trivial trading equilibrium. We examine three possible equilibria for $N_I = 310$, the Reduced Trade, the Skewed Buy, and the Skewed Sell equilibria. In the "hold" equilibrium, we show that the market maker can charge a^L and b^L, the commissions that maximize the liquidity trader losses (total associate profits) at 375. That is, total associate profits are higher for $N_I = 310$ with partial disclosure than for $N_I = 300$ with full disclosure. The bias reduces the *per* associate profit and allows the market makers to lower the ask commission to a^L. In a full disclosure equilibrium, the market makers can reduce per associate profit only by increasing a. The requirement that the partial disclosure equilibrium be subgame perfect induces the non-monotonicity. One could imagine a partial disclosure equilibrium for $267 < N_I \leqslant 300$ that recaptures a^L and b^L. However, if $N_I \leqslant 300 = N_I^*$, the market makers' profit-maximizing a generates $\Pi^A(\cdot) > 0$. Therefore, market shutdown is not credible because each market maker has incentive to reopen the market unilaterally. As a result, a partial disclosure equilibrium is not sustainable for $N_I \leqslant 300$.

5. Endogenous choice of N_I

First, we argue that the optimal choice of associates may exceed N_I^*. Hence, the insider's optimal disclosure strategy will entail partial disclosure. Next, we define and prove the existence of a partial disclosure trading equilibrium in which the insider chooses the number of informed traders. Given the multiplicity of equilibria from the previous section and the generality of the specification of the insider's preferences, it cannot be claimed that this is the insider's optimal partial disclosure strategy. Finally, we restrict the insider's preferences, allowing us to determine the insider's maximum possible utility, and show that even then multiple equilibria are possible.

Any trading equilibrium involves the informed traders sharing the pool of losses provided by the liquidity traders. As the number of informed traders increases, each trader's expected profits are lower. That is, the insider can provide information to more associates only by reducing its value. As a result, the insider's benefit function, $H(N_I, I(a, k^A) + I(b, -k^B))$, involves a tradeoff between its two arguments.

Consider the full disclosure equilibrium with bid and ask commissions of a^* and b^*. These commissions permit the maximum full-disclosure number of clients N_I^*. Given symmetry ($a^* = b^*$) and the market makers break-even condition, the insider's benefit is $H(N_I^*, 2N_L(a^*)/N_I^*)$. Differentiating with respect to N_I yields $B_1 - 2B_2N_L(a^*)/N_I^2$.[14] If the insider places a higher value on providing information

[14] The analysis is informal here. First, increasing N_I from N_I^*, without introducing bias, means the market breaks down. Therefore, the average profit functions are not continuous. Second, N_I is not a continuous variable. Our goal is to provide intuition for the optimality of $N_I > N_I^*$ at minimal expositional cost.

to a large number of associates than he places on the magnitude of each individual associate's profits ($B_1 \gg B_2$), then his utility is increasing in N_I. If sustainable in equilibrium, he chooses $N_I > N_I^*$.

We now define and prove the existence of an equilibrium. The market maker does not know with certainty how many associates the insider will advise. In equilibrium there must be a mechanism to enforce the equilibrium number of associates, N_I^{eq}. Otherwise, the insider would provide information to as many associates as possible since the trading commissions are set prior to the insider informing his associates. Let v represent the market makers' beliefs about the number of associates chosen by the insider.

Definition 3. An incentive compatible partial disclosure trading equilibrium is a set of trading commissions, m, market makers' beliefs (κ^A, κ^B, v), and insider's strategy $([y^{\text{sell}}, y^{\text{buy}}], N_I)$ such that

(i) $N_L L(a) = N_I I(a, \kappa^A)$ and $N_L L(b) = N_I I(b, -\kappa^B)$. The market makers' expected value of current period distortions and number of associates can be formed conditional on previous periods' distortions and the order flow, such that $\kappa^A = k^A$, $\kappa^B = k^B$ and $v = N_I$.
(ii) The insider divides the real line into three intervals such that the insider advises his associates to buy if $\delta_{t+1} \geqslant a - k^A$, sell if $\delta_{t+1} \leqslant -b - k^B$, and hold otherwise.
(iii) $\text{E}[\tilde{\delta}_{t+1}|\text{buy}] \geqslant a$; $-b < \text{E}[\tilde{\delta}_{t+1}|\text{hold}] < a$; $\text{E}[\tilde{\delta}_{t+1}|\text{sell}] \leqslant -b$.

Although market makers cannot identify which traders are associates and which are liquidity traders, they do observe total order flow. If, for example, the total order flow is greater than $N_I^{\text{eq}} + N_L$, the market makers infer that the insider has defected from the equilibrium and shut down the market in all future periods. Similarly, the market can, conditional on the realization of δ_{t+1}, infer a defection from the equilibrium level of bias. Thus, the defection condition for β is a function of deviations from both the equilibrium bias and the total order flow.

Proposition 3. *If the insider's discount rate β is sufficiently high, a subgame perfect symmetric Reduced Trade partial disclosure equilibrium exists in which*:

(i) *The market makers charge* (*associate profit maximizing*) *commissions $m = (b^L, a^L)$ if the order flow in period $t-1$ does not imply* (a) *that the ask distortion was less negative than $-k^A$* (b) *that the positive bid distortion was less positive than k^B and* (c) *that the number of informed traders was greater than N_I^{eq}. Otherwise, the market makers set the trading commissions to ∞.*
(ii) *The insider's strategy is $y = [(-b^L - k^B), (a^L + k^A), N_I^{\text{eq}}]$. The ask distortion, $-k^A$, is implicitly determined by $N_L L(a^L) = N_I^{\text{eq}} I(a^L, -k^A)$; the bid distortion, k^B is determined by $N_L L(b^L) = N_I^{\text{eq}} I(b^L, -k^B)$. The insider sets the number of associates to $N_I^{\text{eq}} > N_I^*$.*

Other equilibria might, depending on the utility function, generate a more favorable balance of number of associates and average profits, so it cannot be claimed that this equilibrium is the optimal partial disclosure equilibrium for the insider. The generality of the benefit function precludes definitive statements about optimality. We can address a couple of special cases, however.

In the first case, the insider cares about aggregate associate profits, not average profits or the number of associates. For example, preferences might be $\lambda N_I[I(a,k^A)+I(b,-k^B)]$. In this case, an equilibrium supporting a^L and b^L maximizes the insider's utility because these commissions maximize the liquidity traders' losses. In equilibrium, the associates share the pool of liquidity trader losses. As the numerical example in the previous section shows, an equilibrium supporting a^L may or may not entail partial disclosure.

In the second special case, the insider cares about aggregate associate profits and the number of associates. A utility function of $\lambda N_I[I(a,k^A)+I(b,-k^B)]+g(N_I)$, with $g'>0$, characterizes such preferences. The insider prefers an equilibrium that supports a^L and b^L for $\bar{N}$ clients, yielding maximal utility $2\lambda N_L L(a^L)+g(\bar{N})$. It is easy to show that such a Reduced Trade equilibrium exists. If the insider chooses the upper bound of associates $\bar{N}$, the market makers' breakeven condition implies that $I(a^L,-k^A)=N_L L(a^L)/\bar{N}$. In the Reduced Trade equilibrium, the insider can drive $I(a^L,-k^A)$ arbitrarily close to 0 by increasing the amount of negative bias. Clearly, then, a level of bias exists such that the preceding equality holds.

The Reduced Trade equilibrium is not the unique equilibrium that generates the maximal utility $2\lambda N_L L(a^L)+g(\bar{N})$. Consider a partial disclosure strategy with noncontiguous regions in which the insider tells his associates to buy only if the realization of δ_{t+1} is in the interval $[a^L, a^L+\varepsilon]$, choosing a symmetric interval for sell signals. For more extreme realizations of the information, he provides no trading advice. Symmetry implies that the associates optimally hold unless told otherwise. Clearly, the insider can drive the average associate profit arbitrarily close to 0 by lowering ε, the size of the information region, thereby replicating the maximal utility. In principle, one could construct an infinite number of equilibria producing the maximal utility.

6. Discussion

We examine the information dissemination strategies of an insider who has private information about the payoff of a security. By providing information to privileged associates (institutional investors or analysts, for example), the insider creates an adverse selection problem for the competitive market makers. If the number of trading cost-sensitive liquidity traders is high relative to the number of associates, the market makers can find break-even trading commissions even if the insider fully discloses his information to his associates. However, if the number of liquidity traders is not high enough, the information asymmetry induced by full disclosure is too severe and no breakeven commission exists for the market makers. In this case, corporate insiders can mitigate the adverse selection problem by biasing and coarsening the information provided to associates.

The bias, or distortion, is necessary to reduce the market makers' trading losses to the informed associates. The coarsening, in the form of trading rules, is necessary to preclude the associates from inferring the true realization of the underlying information.

As an alternative to coarse and biased decision rules, the insider could instead report a noisy version of the information realization to the associate. In principle, such a mechanism could eliminate the adverse selection problem by reducing both the probability of trade and the profit per transaction. Enforcement of such a mechanism in equilibrium would be complex. The market makers can observe ex post the noise added to the underlying information. However, they cannot verify the *distribution* from which the noise was drawn, and it is the distribution upon which the equilibrium must depend. This does not preclude an equilibrium, but complicates it. The deterministic bias in the signal scheme simplifies the enforcement. More importantly, a recommendation scheme can reduce the adverse selection problem without reducing the frequency of trade.

Consider a manager who cares about both supplying information to institutional investors and maintaining a highly liquid market for shares in his firm. These objectives may be at odds. Insiders disclose information to influential analysts or institutional investors, in order to curry favor with them and reap a tangible benefit. For example, the average public company wants to attract analysts and their research coverage along with the investor base that coverage can generate. Since benefits to the insiders are increasing in the number of affiliates to whom they provide information, due to reputation concerns or benefits the insider derives from providing information to others (e.g., positions on boards of directors, greater press coverage), insiders wish to share the information with many affiliates. However, the more affiliates, the greater the information asymmetry and lower the market liquidity. As a result, the insider must "manage" or limit the flow of information to maximize his total objective function.

More specifically, suppose the manager reveals to his large investors the content of an earnings warning (positive or negative) prior to its press release. The warning might take the form: "earnings are expected to fall below Wall Street consensus estimates". In our model, this news would prompt a sale of shares by these informed associates. If the information was perfectly precise, and there were sufficiently many shares traded by associates, the market would collapse. Informed associates would exploit their information, transacting whenever the share price was different from their information, causing the market maker to incur losses. To offset these losses, the market maker would increase bid and ask commissions, but this would ultimately drive out liquidity. Eventually commissions would be so high there would be no trade by any market participants. If instead the manager provides information to his institutional investors with some distortion, institutional investors benefit from heeding the warnings on average. However, the manager will fail to warn his institutional investors despite having information on which they would trade.

Alternatively, consider a manager that cares about analyst following along with maintaining a highly liquid market for shares in their firms. Since analysts are limited in the number of firms they can follow, they are more likely to follow those for which they have information to assist them in making forecasts and recommendations. On one hand, higher analyst following might attract attention for the firm and increase liquidity. On the other hand, because financial analysts will offer their private information to select clients first, it increases information asymmetry. Again, the managers are faced with limiting the flow of information.

With information about a forthcoming earnings warning, analysts can update their forecasts and recommendations and call their clients, who then expect to profit on this information prior to the public announcement. On average, the information from management is valuable, and thus correcting estimates is rational. However, in some cases, the adjustment to the analysts' estimates is too severe, and the firm "surprises" the market at the earnings announcement date. "You see this all the time on the Street—a company will report some bad news weeks or days before earnings are announced, and analysts will reduce estimates. Then lo and behold, a company will end up beating estimates".[15]

We have modeled the market environment in a stylized way, but our results are robust to a number of the assumptions. For example, although we focus on short lived information, this is not essential to the model. Suppose there were fixed periods of trade, but information was not revealed until the end of several of these trading rounds. The market maker, after observing actual volume in the first trading round could adjust his expectation of value. However, information sellers are faced with a problem that differs from the first round *only* by the market makers' quoted bid and ask prices. Additionally, we model competitive market makers instead of a monopolist market maker. The competition between market makers eliminates market-maker profits and allows a simple characterization of the equilibrium bid-ask prices. However, the condition for market failure does not depend on the number of market makers. Unless there is a set of finite bid and ask prices that yield zero profits (i.e., in which liquidity traders participate), there will not be a non-trivial trading equilibrium regardless of the number of market makers. We also assume that the insider receives perfect information. Noisy information would reduce the magnitude of the bias necessary to support a trading equilibrium. Finally, we assume that the insider does not trade on his own information. As long as the insider is restricted in the number of shares he trades (as are the other market participants), he will face the same tradeoffs between number of associates and precision of the information he discloses. The model does not address competition between multiple insiders.

Appendix

Proof of Proposition 1. Clearly, at N_I^* it must be the case that the equilibrium ask commission maximizes $\Pi^A(a)$ and that the maximized value of $\Pi^A(a) = 0$. Otherwise, one could increase N_I. Let a^* be the equilibrium ask commission at

[15] Business Week, "Ho-Hum, Another earnings surprise: investors now look for companies to routinely beat profit estimates", May 24, 1999.

N_I^*. That is, $\Pi^A(a^*) = N_L L(a^*) - N_I^* I(a^*) = 0$. Therefore, for $N_I < N_I^*$, $\Pi^A(a^*) = N_L L(a^*) - N_I I(a^*) > 0$. Because $\Pi^A(0) = N_L L(0) - N_I I(0) < 0$ $\forall N_I$ and $\Pi^A(\cdot)$ is a continuous function in a, there exists a such that $\Pi^A(a) = N_L L(a) - N_I I(a) = 0$ for $0 \leqslant N_I \leqslant N_I^*$.

It is easy to see that the market makers' and insider's strategies are optimal. With multiple market makers, traders will transact with the market maker with the lowest bid and ask commissions. Bertrand competition drives the market makers' profits to zero, and thus equilibrium trading bid and ask commissions will be the lowest values which earn zero profits. Since full disclosure recommendations lead to optimal (associate profit maximizing) decisions, the insider has no incentive to deviate from his full disclosure strategy. Similar analysis holds for the bid side. □

Proof of Observation 2. Let $k^{A'} > k^A$.

$$\begin{aligned} I(a,k^A) &= \int_{a-k^A}^{a} (\delta - a) f(\delta)\,\mathrm{d}\delta + \int_{a}^{\bar{\delta}} (\delta - a) f(\delta)\,\mathrm{d}\delta \\ &> \int_{a-k^A}^{a} (\delta - a) f(\delta)\,\mathrm{d}\delta + \int_{a}^{\bar{\delta}} (\delta - a) f(\delta)\,\mathrm{d}\delta + \int_{a-k^{A'}}^{a-k^A} (\delta - a) f(\delta)\,\mathrm{d}\delta \\ &= I(a,k^{A'}) \end{aligned}$$

Since $I(a,k^A)$ is decreasing in k^A, $\Pi^A(a,k^A)$ is increasing in k^A. □

Proof of Observation 3.

$$\begin{aligned} \lim_{k^A \to a-\underline{\delta}} \int_{a-k^A}^{\bar{\delta}} (\delta - a) f(\delta)\,\mathrm{d}\delta \to \int_{\underline{\delta}}^{\bar{\delta}} (\delta - a) f(\delta)\,\mathrm{d}\delta \\ = \int_{\underline{\delta}}^{\bar{\delta}} \delta f(\delta)\,\mathrm{d}\delta - \int_{\underline{\delta}}^{\bar{\delta}} a f(\delta)\,\mathrm{d}\delta \\ = \mathrm{E}[\delta] - a \\ = -a < 0. \end{aligned}$$

Observe also that $I(a,0) = \int_a^{\bar{\delta}} (\delta - a) f(\delta)\,\mathrm{d}\delta > 0$. Since $I(a,k^A)$ is continuous in k^A, the Intermediate Value Theorem implies that there exists $k^{A''}$ such that $I(a,k^{A''}) = \varepsilon > 0$, ε arbitrarily small. Since we assume that $N_L L(a) - N_I I(a,0) > 0$, this implies that for some k^A, $\Pi^A = 0$. □

Proof of Observation 4.

$$\lim_{k^B \to -b-\underline{\delta}} \int_{\underline{\delta}}^{-b-k^B} (-b - \delta) f(\delta)\,\mathrm{d}\delta \to \int_{\underline{\delta}}^{\underline{\delta}} (b - \delta) f(\delta)\,\mathrm{d}\delta = 0.$$

We know that $I(b,0) > 0$. (Recall that by symmetry $I(b,-k^B)$ are the associate's profits if there is *positive* distortion in the sell recommendation.) Since $I(b,-k^B)$ is continuous in $-k^B$, the Intermediate Value Theorem implies that there exists $-k^{B'}$

such that $I(b, -k^{B'}) = \varepsilon > 0$, ε arbitrarily small. Since we assume that $N_L L(b) - N_I I(b, 0) > 0$, this implies that for some $-k^B$, $\Pi^B = 0$. $\square$

Proof of Proposition 2 (Reduced Trade Equilibrium). *Insider's strategy*: Holding the market makers' pricing policy fixed we consider the insider's strategy. If the insider biases by $-k^A$ and k^B in every period, then the market maker will charge commissions of $a_t = a^*$ and $b_t = b^*$ in each period. The insider's utility on the equilibrium path is

$$\sum_{t=1}^{\infty} \beta^{t-1} H(N_I, I(a^*, -k^A) + I(b^*, -k^B)) = \frac{H(N_I, I(a^*, -k^A) + I(b^*, -k^B))}{1-\beta}.$$

The insider's possible defection region (on the buy side) is $\delta_{t+1} \in (a^*, a^* + k^A]$. If $0 \leqslant \delta_{t+1} \leqslant a^*$ or $a^* + k^A < \delta_{t+1}$, the full disclosure decision rule (hold or buy, respectively) is the same as the partial disclosure rule. The most profitable defection for the insider is for a realization of $\delta_{t+1} = a^* + k^A$. In this case, defection entails a buy signal instead of a hold. Each associate nets a profit of k^A. The expected utility for the insider from such a defection is

$$H(N_I, k^A) + p \times 0 + (1-p)\frac{\beta}{1-\beta} H(N_I, I(a^*, -k^A) + I(b^*, -k^B)).$$

In this expression, p is the probability that the total number of buy orders exceeds the number of liquidity traders given that the associates buy. If this happens, the market maker knows with certainty that the insider defected from the equilibrium. In mathematical terms, p is

$$p \equiv 1 - \sum_{x=0}^{N_L - N_I} \binom{N_L}{x} [\Pr(a^* < r)]^x [1 - \Pr(a^* < r)]^{N_L - x}.$$

If the insider does not defect, his expected utility is

$$H(N_I, 0) + \frac{\beta}{1-\beta} H(N_I, I(a^*, -k^A) + I(b^*, -k^B)).$$

Therefore, the insider will not defect if

$$H(N_I, k^A) - H(N_I, 0) < p \frac{\beta}{1-\beta} H(N_I, I(a^*, -k^A) + I(b^*, -k^B)).$$

Clearly there exists β such that the insider will not defect. If he will not defect for the most profitable defection at $\delta_{t+1} = a^* + k^A$, he will not defect for less extreme realizations of δ_{t+1} either.

Market makers' strategy and expectations. Holding the insider's strategy fixed with levels of bias k^A and k^B in each period t, a competitive market maker must select the lowest trading commissions, that yield zero profits. Since a^* maximizes the market makers' profit condition with zero bias, and k^A is precisely the amount of bias required for the market maker to break even, any lower trading commission will result in negative profits for the market maker. Higher trading commissions are eliminated by the Bertrand competition between market makers. The market makers' expectation of buy (sell) side bias of k^A (k^B) if they do not infer that last period's buy and sell side biases were less than k^A and k^B respectively are confirmed on the equilibrium path. Off the equilibrium path, consider bias $k_t^A < k^A$ or $k_t^B < k^B$. In period t, the market makers earn negative expected profits and shut down. The shut down is sustained by off-equilibrium path beliefs $\mathrm{E}[k_{t+1}^A | k_t^A < k^A \cup k_t^B < k^B] = 0$ and $\mathrm{E}[k_{t+1}^B | k_t^A < k^A \cup k_t^B < k^B] = 0$; the only trading commissions that the market makers can charge in period $t+1$ and earn non-negative profits is in the interval $[\bar{\delta}, \infty)$. $\square$

Proof of Proposition 3 (Reduced Trade Equilibrium). Similar to the proof to Proposition 2, we will derive a discount rate β such that the insider does not defect. Because there are two dimensions to defection, bias and number of associates, the proof is more involved. If the realization of δ_{t+1} is greater than $a^L + k^A$, both full and partial disclosure require the insider to advise buy. Therefore, the insider can defect only along the associate size dimension. If $a^L \leqslant \delta_{t+1} < a^L + k^A$, the insider can defect along both dimensions. We will discuss these cases in separate subsections.

Defection with respect to number of associates only. Both the current-period defection profits *and* the probability of detection are increasing in the number of associates. A priori one cannot determine the insider's optimal defection. A defection to $N_I = \bar{N}$ is not necessarily the optimal defection. The proof strategy will be to show that for any $\hat{N}_I \in [N_I^{\mathrm{eq}}, \bar{N}]$, there exists a β such that the insider plays the equilibrium strategy of Hold rather than instructing $\hat{N}_I$ associates to buy, even with a realization of $\delta_{t+1} = \bar{\delta}$. The expected value along the equilibrium path is

$$\frac{H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B))}{1-\beta}.$$

The expected utility from a defection to $\hat{N}_I$ associates at $\delta_{t+1} = \bar{\delta}$ is

$$H(\hat{N}_I, \bar{\delta} - a^L) + (1-p)\frac{\beta}{1-\beta} H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B)),$$

where p, the probability of detection, is

$$p \equiv \sum_{x=1+N_I^{\mathrm{eq}}+N_L-\hat{N}_I}^{N_L} \binom{N_L}{x} [\Pr(a^L < r)]^x [1 - \Pr(a^L < r)]^{N_L - x}.$$

If the insider does not defect, his expected utility is

$$H(N_I^{\mathrm{eq}}, \bar{\delta} - a^L) + \frac{\beta}{1-\beta} H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B)).$$

Therefore, the insider will not defect if

$$H(\hat{N}_I, \bar{\delta} - a^L) - H(N_I^{\mathrm{eq}}, \bar{\delta} - a^L) < p\frac{\beta}{1-\beta} H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B)).$$

Clearly there exists β such that the insider will not defect. If he will not defect for the most profitable defection at $\delta_{t+1} = \bar{\delta}$, he will not defect for less extreme realizations of δ_{t+1} either. Each $\hat{N}_I$ in the range generates a new β. Let the maximum of this set be β^N.

Defection with respect to both number of associates and bias. If $a^L < \delta_{t+1} < a^L + k^A$, the insider can defect both with respect to the number of associates and the bias. The current-period defection profits are increasing in the size of the bias defection (the distance between the realization of $\delta_{t+1} > a^L$ and a^L), but the probability of detection is fixed with respect to the size of the defection. Thus, we need consider only the maximum defection (instruct buy for the realization of $\delta_{t+1} = a^L + k^A$). Because both the current-period defection profits *and* the probability of detection are increasing in the number of associates, the proof strategy will be to show that for any $\hat{N}_C \in [N_I^{\mathrm{eq}}, \bar{N}]$, there exists a β such that the insider plays the equilibrium strategy of instructing N_I^{eq} associates to hold rather than instructing $\hat{N}_I$ associates to buy at $\delta_{t+1} = a^L + k^A$. The expected utility for the insider from such a defection is

$$H(\hat{N}_I, k^A) + (1-p)\frac{\beta}{1-\beta}H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B)),$$

where

$$p \equiv 1 - \sum_{x=0}^{N_L - \hat{N}_I} \binom{N_L}{x} [\Pr(a^L < r)]^x [1 - \Pr(a^L < r)]^{N_L - x}.$$

If the insider does not defect, his expected utility is

$$H(N_I^{\mathrm{eq}}, 0) + \frac{\beta}{1-\beta}H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B)).$$

Therefore, the insider will not defect if

$$H(\hat{N}_I, k^A) - H(N_I^{\mathrm{eq}}, 0) < p\frac{\beta}{1-\beta}H(N_I^{\mathrm{eq}}, I(a^L, -k^A) + I(b^L, -k^B)).$$

Clearly there exists β such that the insider will not defect. If he will not defect for the most profitable defection at $\delta_{t+1} = a^L + k^A$, he will not defect for less extreme realizations of δ_{t+1} either. Each $\hat{N}_I$ in the range generates a new β. Let the maximum of the set be β^K. The discount rate necessary for the proposition is $\mathrm{Max}\{\beta^N, \beta^K\}$.

Market makers' strategy and expectations: Since a^L and b^L maximize the gains from trading with liquidity traders, market maker profits are strictly decreasing for lower commissions. The biases k^A and k^B allow the market makers to break even, and any higher commissions are eliminated by Bertrand competition. Hence, the market makers choose a^L and b^L on the equilibrium path. The following beliefs support the market makers' off equilibrium path actions (market closure):

$$\mathrm{E}[k_{t+1}^A | k_t^A < k^A \cup k_t^B < k^B \cup \text{orderflow} > N_{C,t} + N_L] = 0,$$

$$\mathrm{E}[k_{t+1}^B | k_t^A < k^A \cup k_t^B < k^B \cup \text{orderflow} > N_{C,t} + N_L] = 0$$

and

$$\mathrm{E}[N_{\mathrm{C},t+1}|k_t^A < k^A \cup k_t^B < k^B \cup \text{orderflow} > N_{\mathrm{C},t} + N_L] = \bar{N}.$$

Given these beliefs, the market makers cannot break even post-defection unless they set the commissions to ∞. $\square$

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